EFFECT OF MASS TRANSFER ON THE PROCESS OF HEAT TRANSFER WITH SUBLIMATION IN A RAREFIED GAS MEDIUM FOR COUETTE FLOW

P. A. Novikov and B. M. Smol'skii

The effect of transverse mass flow on heat transfer has been determined for the case of sublimation of a moving plate in a rarefied gas medium. It has been established that at Knudsen numbers Kn > 0.001 it is necessary to take into account the effect of the wall layer near the subliming surface.

Film cooling is a special method of protecting the components of jet engines, the walls of combustion chambers, etc. Since the surface film is disturbed by the flow of heat transfer agent, the efficiency of film cooling diminishes in the direction of the gas flow. This disadvantage can be avoided by using a new thermal shielding process, evaporative cooling. For this purpose it is possible to use the process of transpiration of a liquid through a porous element or the method of sublimation. In this case the layer of vapor or sublimate is continuously renewed, and there are no local discontinuities along the cooled surface. In this method of cooling the flow of coolant vapor from the surface of the element reduces the heat transfer coefficient as compared with heat transfer conditions in the absence of evaporative cooling.

In order to obtain a relation that would approximately describe this reduction in the heat transfer rate it is possible to substitute a simplified model for the actual process. As this model we will use the Couette flow between two parallel walls, one of which is stationary while the other moves at constant velocity in its own plane.

We will consider the flow in a coordinate system with axes x and y for sublimation of one of the plates, as shown in Fig. 1.

In our case plate 2 is in motion and sublimes at a rate v. In this case the temperature field between the plates will differ from the case of flow without sublimation. At constant pressure in the directions x and y and constant velocity v along the plate the temperature T will vary only in the direction of the y axis, the rate v being constant in the y direction if the thermodynamic parameters of the medium are constant. Since gasdynamic effects are much more complicated in rarefied gases than in dense media, owing to attenuation of the interaction between the gas and the wall, the no-slip condition of continuous flow theory ceases to be completely valid. During the flow process the rarefied gas begins to slip close to the wall, forming a wall layer whose molecules interact with the wall according to a law that differs from continuum flow. On average, the gas (vapor) molecules pass through this wall layer without colliding, the most complicated problem being that of studying the reflection of gas molecules from the wall. An exact analysis of the interaction between the gas and the wall surface re-

quires a knowledge of the distribution function in the layer of gas directly adjacent to the wall. The thickness of this layer may be taken equal to the mean free path of the molecules (Knudsen layer). This layer always exists but in a continuous flow is so thin that it can be neglected. As the total pressure falls, the Knudsen layer becomes thicker and plays an ever-increasing part in the process of heat and mass transfer.
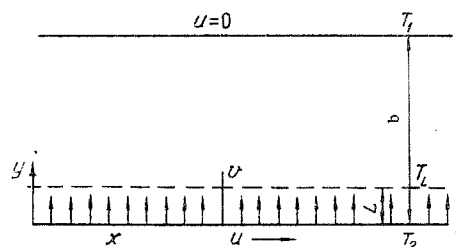

Fig. 1. Couette flow diagram for sublimation of a moving plate.

Therefore the presence of a wall layer creates difficulties both in formulating the boundary conditions and in selecting a model of the mechanism of heat transfer through the wall layer to the wall.

If the Knudsen parameter is small, e.g., Kn < < 0.001, the gas may be regarded as a continuum up to the wall itself. Otherwise, i.e., when Kn > 0.001, it is not desirable to neglect the effect of the wall layer.

We will calculate the heat flow from surface 1 to the subliming surface 2. Since the temperature in the flow field is a function of y only, in our case the temperature field between the plates can be obtained by solving the differential equation

$$\lambda \frac{d^2 T}{dy^2} - \rho v c \frac{dT}{dy} = 0. \tag{1}$$

The boundary conditions with allowance for the presence of a wall layer are taken in the following form:

$$y = L, \; T = T_L; \quad y = b, \; T = T_1.$$

Then the solution of this equation will have the form

$$\Theta = \frac{\exp\left(\dfrac{\rho v c}{\lambda} y\right) - \exp\left(\dfrac{\rho v c}{\lambda} L\right)}{\exp\left(\dfrac{\rho v c}{\lambda} b\right) - \exp\left(\dfrac{\rho v c}{\lambda} L\right)}. \tag{2}$$

Introducing dimensionless complexes, we obtain

$$\Theta = \frac{\exp\left(y/b \; \mathrm{Re}_v \mathrm{Pr}\right) - \exp\left(\mathrm{Re}_v \mathrm{Pr} \, \mathrm{Kn}\right)}{\exp\left(\mathrm{Re}_v \mathrm{Pr}\right) - \exp\left(\mathrm{Re}_v \mathrm{Pr} \, \mathrm{Kn}\right)}. \tag{3}$$
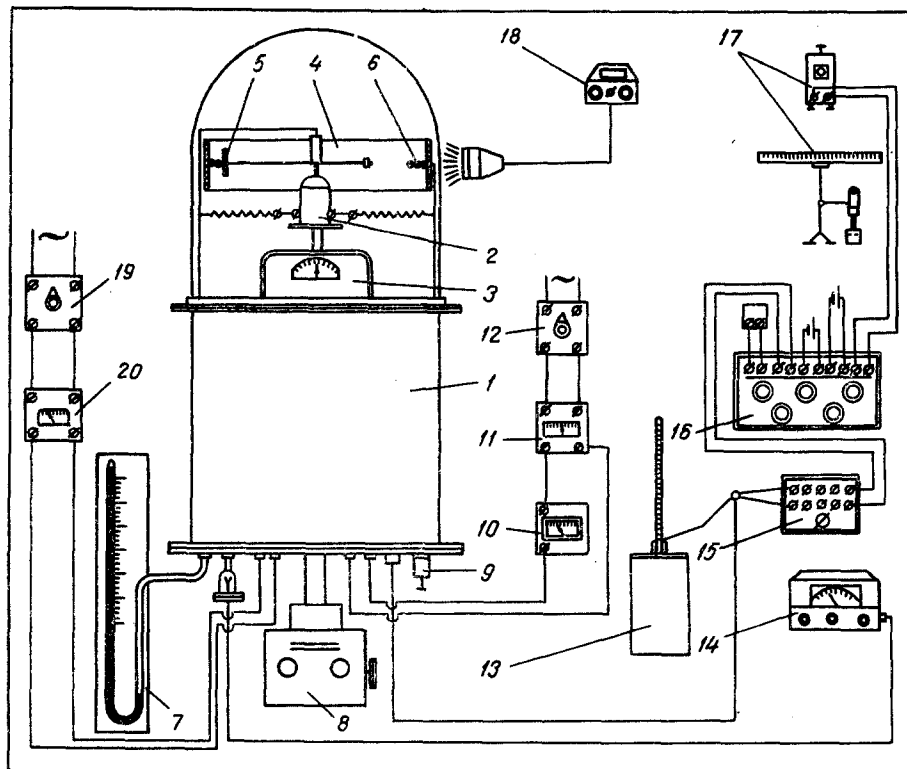
Fig. 2. Schematic of experimental apparatus.
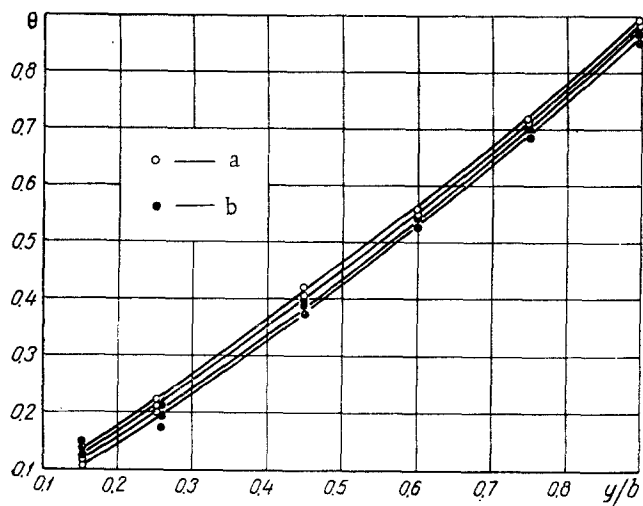

Fig. 3. Dimensionless temperature profile between plates in a Couette flow: a) calculated values, b) experimental data at p = $= 1.33 \cdot 10^3$ N/m^2.

On the basis of the molecular-kinetic theory of gases Grad [6] established that the statistical gas temperature is proportional to the kinetic energy of motion of molecules with corresponding mass flow velocity u. In this case the statistical temperature of the gas at a distance L from the plate can be determined from the equation

$$T_L = T_m \left[1 + \left(1 - \frac{\alpha_A}{1+\alpha_A}\right) \sqrt{\frac{T_2}{T_m}} \right] \times$$

$$\times \left[1 + \left(1 - \frac{\alpha_A}{1+\alpha_A}\right) \sqrt{\frac{T_m}{T_2}} \right]^{-1} +$$

$$+ T_m \frac{1}{12} \frac{u^2}{RT_m} (1 - \alpha_A^2) \sqrt{\frac{T_m}{T_2}} \times$$

$$\times \left[1 + \left(1 - \frac{1-\alpha_A}{2}\right) \left(\sqrt{\frac{T_m}{T_2}} - 1\right)^2 \right]^{-1}. \quad (4)$$

The heat flow to the wall layer at a distance L from the subliming surface to the plate is determined from the expression

$$q = -\lambda_{mix} \left| \frac{dT}{dy} \right|_{y=L} = \frac{-\lambda_{mix}(T_1 - T_L) Re_v Pr}{b \{\exp [Re_v Pr (1 - Kn)] - 1\}}. \quad (5)$$

The thermal conductivity λ_{mix} of the vapor-air mixture between the heater and the wall layer can be found from the equation

$$\lambda_{mix} = \frac{\lambda_1}{1 + A_{12}(x_2/x_1)} + \frac{\lambda_2}{1 + A_{21}(x_1/x_2)}. \quad (6)$$

In the presence of a temperature jump in the wall layer we get an additional resistance to heat transfer; therefore we can write an expression for the over-all convective heat transfer coefficient including the thermal resistances of both the main flow and the temperature jump, i.e.,

$$q = \alpha (T_1 - T_2). \quad (7)$$

It is assumed that the amount of heat transferred to the wall layer is equal to the heat supplied to the subliming surface of the plate.
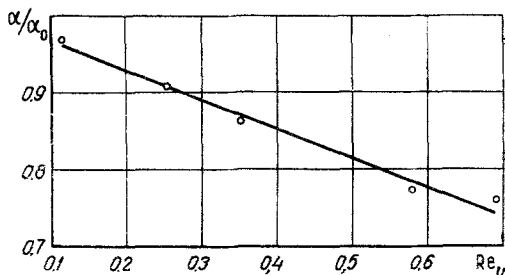

Fig. 4. Experimental relation between relative transfer coefficient and dimensionless rate of transverse flow of subliming substance.

If we compute the heat transfer coefficient α from Eqs. (5) and (7)

$$\alpha = \frac{-\lambda_{mix}(T_1 - T_L) Re_v Pr}{b(T_1 - T_2) \{\exp [Re_v Pr (1 - Kn)] - 1\}} \quad (8)$$

and compare it with the coefficient α_0 in a Couette flow without sublimation at the same thermodynamic parameters of the medium, we find that the coefficient with sublimation is less than the pure heat transfer coefficient, i.e., $\alpha < \alpha_0$:

$$\frac{\alpha}{\alpha_0} = \frac{-\lambda_{mix}(T_1 - T_L) Re_v Pr}{\lambda_b (T_1 - T_2) \{\exp [Re_v Pr (1 - Kn)] - 1\}}, \quad (9)$$

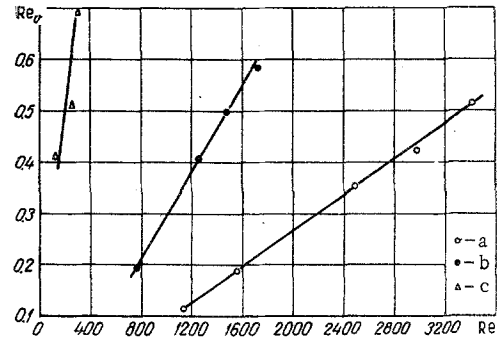where α_0 is taken equal to λ_b/b.

Fig. 5. Dimensionless relation between transverse mass flow rate and the velocity of the subliming plate at different ambient pressures: 1) $1.33 \cdot 10^4$ N/m^2; b) $6.65 \cdot 10^4$; c) $1.33 \cdot 10^3$.

In order to verify the correctness of these relations we conducted experiments relating to Couette flow.

The experiments were performed on a vacuum apparatus (Fig. 2) consisting of a metal cylinder 1, closed at the top by a glass bell. Inside the chamber we set up an electric balance 3, mounting a dc motor 2 on the pan. To the shaft of the electric motor we attached copper and constantan thermocouple rings and a balance arm (the thin tube from a hypodermic needle). The rings were made of constantan wire, which formed a junction with a copper wire at the temperature measuring point. A textolite sleeve was fitted over the rings to hold the thermocouple contacts. To make the contacts we used wire of the same cross section as in the thermocouple working junction. The thermocouples were led out from the contacts to the temperature measuring points through the hollow balance arm, at one end of which there was attached a short plate 5 covered with the subliming material (a hydrocarbon, naphthalene) and at the other a bob weight. Inside the glass bell we set up a metal ring 4, whose temperature could be regulated within desired limits by means of an internal electric heater. The outside and upper and lower faces of the metal ring were thermally insulated. The temperature of the inside surface of the ring could be kept constant. The heater voltage was regulated by means of an auto-transformer 12. The variation of heater power could be checked by means of a voltmeter 11 and an ammeter 10. The velocity of the subliming plate was regulated by means of a rheostat 19, whose voltage was checked by means of voltmeter 20. The speed of the

electric motor was determined by means of a strobo-scope 18. The pressure in the chamber was kept constant by means of a vacuum pump 8 and regulated with a needle valve 9. The total pressure was measured with a mercury U-manometer 7 and a thermocouple vacuum gauge 14. The temperature field between the subliming plate and the inside surface of the ring was measured with copper-constantan thermocouples 6.

The temperature measuring circuit included the following elements: mirror galvanometer 17, low-resistance potentiometer 16, thermocouple switch 15, vacuum bottle with thermometer for the cold junction of the thermocouples 13, contact system, etc.

The rate of mass transfer was determined from the change of mass of the substance subliming from the surface of the plate.

We will now discuss the results of the experiments.

In Fig. 3 we have plotted the temperature profile between the plates from data calculated from Eq. (3) for various values of Re_v. For comparison we also show the temperature distribution obtained experimentally. As may be seen from the graph, at the same thermodynamic parameters of the medium, as the transverse mass flux increases the temperatures of the surface of the plate and the vapor-air mixture between the plates decrease.

According to temperature measurements near the subliming plate, in reality the temperature curve is sharply distorted near the wall. This behavior of the curve is attributable to the fact that the distribution of gas molecules in the incident flow is significantly changed as a result of collision with the vapor molecules escaping from the plate. This distribution cannot be expressed exactly using ordinary transport theory, since in this case the accommodation coefficient α_A depends not only on temperature but also on the velocity and density of the molecules striking the wall or escaping from it. The temperature accommodation varies sharply with the properties of the gas (vapor) and the nature of the subliming wall. If the wall is smooth, the impinging gas molecules are reflected from it at first impact and the accommodation is weak. In the case of a rough wall, which corresponds to the case of sublimation, on striking the wall the gas molecules interact with it for a certain time and the accommodation is more complete.

It is clear from Fig. 4 that as the transverse mass flow rate increases the heat transfer coefficient decreases. In our case the parameter Re_v varied in the range 0.11—0.69, the relative coefficient varying correspondingly from 0.97 to 0.76.

If we compare the values of α/α_0 obtained from (9) with the experimental data, we obtain quite satisfactory agreement.

It is clear from Fig. 5 that as the total pressure decreases the transverse mass flow rate sharply increases, the total pressure having a stronger effect on the rate of separation of vapor from the surface of the material than the velocity of the plate.

At pressures $p < 1.33 \cdot 10^3$ N/m^2 during periods of very intense sublimation there is a change in the heat transport mechanism due to disturbance of the

wall layer by local evaporation processes. Thus, for example, in our experiments on the sublimation of naphthalene at a dimensionless transverse flow rate $Re_v > 0.8$ we obtained an intensification of the heat transfer process, the coefficient in the presence of sublimation being greater than the pure heat transfer coefficient. This indicates that the heat transfer rate is significantly affected by the way in which the vapor separates from the sublimation surface, i.e., by the vapor flow regime near the subliming surface of the material.

NOTATION

$v = q_m/\rho$ is the transverse mass flow rate; q_m is the sublimation rate; ρ is the density of vapor-air mixture; c is the specific heat of vapor-air mixture; L is the thickness of wall layer; b is the width of the gap between plates; $Re_v = vb/\nu$ is the Reynolds number calculated from the transverse mass flow rate; ν is the kinematic viscosity of the vapor-air mixture; $Kn = l/L$ is the Knudsen number; l is the mean free path of molecules; Pr is the Prandtl number; $T_m = (3T_2 + T_1)/4$ is the average main-flow temperature outside the wall layer; u is the plate velocity; R is the gas constant;

$$A_{12} = \frac{1}{4} \left\{ 1 + \left[\frac{\mu_1}{\mu_1} \left(\frac{M_2}{M_1} \right)^{0.75} \frac{1 + S_1/T}{1 + S_2/T} \right]^{0.5} \right\}^2 \frac{1 + S_{12}/T}{1 + S_1/T};$$

λ_b is the thermal conductivity of vapor-air mixture between plates; λ_1 and λ_2 are the thermal conductivities of pure components of vapor-air mixture; x_1 and x_2 are the mole fractions of the corresponding components; μ_1 and μ_2 are the viscosities of the pure components; M_1 and M_2 are the molecular weights of pure components; T is the temperature; S_1 and S_2 are the Sutherland constants for the pure components ($S \approx \approx 1.5T_b$, where T_b is the normal boiling point); $S_{12} = \sqrt{S_1 S_2}$ is the Sutherland constant for the mixture; $Re = ub/\nu$ is the main-flow Reynolds number; $\Theta = = (T - T_L)/(T_1 - T_L)$ is the dimensionless temperature parameter.

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